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Non-leptonic B-decays, CP violation & the UT*

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We study the implication of the time-dependent CP asymmetry in $B \to \pi^+\pi^-$ decays on the extraction of weak phases taking into account the precise measurement of $\sin 2\beta$, obtained from the "gold-plated" mode $B \to J/\psi K_S$. Predictions and uncertainties for the hadronic parameters are investigated in QCD factorization. Furthermore, independent theoretical and experimental tests of the factorization framework are briefly discussed. Finally, a model-independent bound on the unitarity triangle from CP violation in $B \to \pi^+\pi^-$ and $B \to J/\psi K_S$ is derived.

1. Introduction

In the standard model (SM), the only source of CP violation is the Kobayashi-Maskawa phase [1], localized in the Unitarity Triangle (UT) of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1,2]. Thanks to the precise measurements at the current B-factories, CP violation could be established in $B_d \to J/\psi K_S$ [3,4], leading to a precise measurement of $\sin 2\beta$, where the current world average yields [5] $\sin 2\beta = 0.739 \pm 0.048$. The extractions of the other two angles α and γ are expected mainly through CP violation in the charmless B decays, such as $B_d \to \pi\pi$ and similar modes [6]. The current B-factories measurements have been averaged to yield [5]:

$$S_{\pi\pi} = -0.74 \pm 0.16, \qquad C_{\pi\pi} = -0.46 \pm 0.13.$$

On the theoretical side, the analysis is challenging due to the need to know the ratio of penguin-to-tree amplitude contributing to this process. In this talk, we present the result of [7,8], where a transparent method of exploring the UT through the CP violation in $B \to \pi^+\pi^-$, combined with the "gold-plated" mode $B_d \to J/\psi K_S$ has been proposed. A model independent lower bound on the CKM parameters as functions of $S_{\pi\pi}$ and $\sin 2\beta$ is derived. Our estimate of the hadronic parameters are carried out in QCD factorization (QCDF) and confronted to other approaches.

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2. Basic Formulas

The time-dependent CP asymmetry in $B \to \pi^+\pi^-$ decays is defined by

$$A_{CP}^{\pi\pi}(t) = -S_{\pi\pi}\,\sin(\Delta m_B t) + C_{\pi\pi}\,\cos(\Delta m_B t), \quad (1)$$

where $$S_{\pi\pi} = \frac{2\,\mathrm{Im}\xi}{1+|\xi|^2}, \quad C_{\pi\pi} = \frac{1-|\xi|^2}{1+|\xi|^2}, \quad (2)$$

with $\xi = e^{-2i\beta}\frac{e^{-i\gamma}+P/T}{e^{+i\gamma}+P/T}$, and β and γ are CKM angles which are related to the Wolfenstein parameters $\bar\rho$ and $\bar\eta$ in the usual way [9].

The penguin-to-tree ratio P/T can be written as $P/T = re^{i\phi}/\sqrt{\bar\rho^2+\bar\eta^2} \equiv re^{i\phi}/R_b$. The real parameters r and ϕ defined in this way are pure strong interaction quantities without further dependence on CKM variables.

For any given values of r and ϕ a measurement of $S_{\pi\pi}$ and $C_{\pi\pi}$ defines a curve in the $(\bar\rho, \bar\eta)$-plane, expressed respectively through

$$S_{\pi\pi} = \frac{2\bar\eta[R_b^2 - r^2 - \bar\rho(1-r^2) + (R_b^2-1)r\cos\phi]}{((1-\bar\rho)^2+\bar\eta^2)(R_b^2+r^2+2r\bar\rho\cos\phi)} \quad (3)$$

and $$C_{\pi\pi} = \frac{2r\bar\eta\,\sin\phi}{R_b^2+r^2+2r\bar\rho\cos\phi}. \quad (4)$$

The penguin parameter $r\,e^{i\phi}$ has been computed in [10] in the framework of QCDF. The result can be expressed in the form

$$r\,e^{i\phi} = -\frac{a_4^c + r_\chi^\pi a_6^c + r_A[b_3+2b_4]}{a_1 + a_4^u + r_\chi^\pi a_6^u + r_A[b_1+b_3+2b_4]}, \quad (5)$$

where we neglected the very small effects from electroweak penguin operators. A recent analysis gives [7,8]

$$r = 0.107 \pm 0.031, \qquad \phi = 0.15 \pm 0.25, \tag{6}$$

where the error includes an estimate of potentially important power corrections. In order to obtain additional insight into the structure of hadronic B-decay amplitudes, it will be also interesting to extract these quantities from other B-channels, or using other methods. In this perspective, we have considered them in a simultaneous expansion in $1/m_b$ and $1/N_C$ (N_C is the number of colours) in (5). Expanding these coefficients to first order in $1/m_b$ and $1/N_C$ we find that the uncalculable power corrections b_i and $H_{\pi\pi,3}$ do not appear in (5), to which they only contribute at order $1/m_b N_C$. Using our default input parameters, one obtains the central value [7]: $(r_{N_C}, \phi_{N_C}) = (0.084, 0.065)$, which seems to be in a good agreement with the standard QCDF framework at the next-to-leading order.

As a second cross-check, one can extract r and ϕ from $B^+ \to \pi^+\pi^0$ and $B^+ \to \pi^+ K^0$, leading to the central value [7] $(r_{SU3}, \phi_{SU3}) = (0.081, 0.17)$, in agreement with the above results[2], although their definitions differ slightly from (r, ϕ) (see [7] for further discussions).

3. UT through CP violation observables

It is possible to fix the UT by combining the information from $S_{\pi\pi}$ with the value of $\sin 2\beta$, well known from the "gold-plated" mode $B \to J/\Psi K_S$. The angle β of the UT is given by

$$\tau \equiv \cot\beta = \sin 2\beta \left(1 - \sqrt{1 - \sin^2 2\beta}\right)^{-1}. \tag{7}$$

The current world average [5] $\sin 2\beta = 0.739 \pm 0.048$, implies $\tau = 2.26 \pm 0.22$. Given a value of τ, $\bar\rho$ is related to $\bar\eta$ by $\bar\rho = 1 - \tau\,\bar\eta$. The parameter $\bar\rho$ may thus be eliminated from $S_{\pi\pi}$ in (3), which can be solved for $\bar\eta$ to yield

$$\bar\eta = \frac{1}{(1+\tau^2) S_{\pi\pi}} \left[\tilde{S}(1 + r\cos\phi) - \sqrt{(1 - S_{\pi\pi}^2)(1 + r^2 + 2r\cos\phi) - \tilde{S}^2 r^2 \sin^2\phi} \right], \tag{8}$$

[2]one can compare also r_{SU3} to its experimental value $r_{SU3}^{exp} = 0.099 \pm 0.014$.

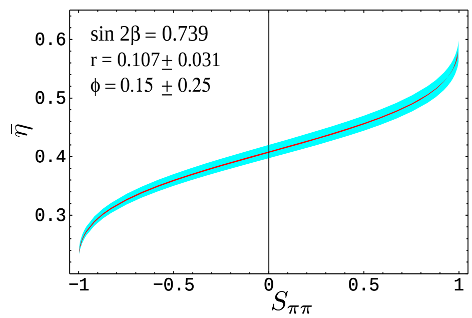

Figure 1. *CKM phase $\bar\eta$ as a function of $S_{\pi\pi}$. The dark (light) band reflects the theoretical uncertainty in the parameter ϕ (r).*

with $\tilde{S} = (1 + \tau S_{\pi\pi})$ The two observables τ (or $\sin 2\beta$) and $S_{\pi\pi}$ determine $\bar\eta$ and $\bar\rho$ once the theoretical penguin parameters r and ϕ are provided.

The determination of $\bar\eta$ as a function of $S_{\pi\pi}$ is shown in Fig. 1, which displays the theoretical uncertainty from the penguin parameters r and ϕ in QCDF. Since the dependence on ϕ enters in (8) only at second order, it turns out that its sensitivity is rather mild in contrast to r. In the determination of $\bar\eta$ and $\bar\rho$ described here discrete ambiguities do in principle arise, however they are ruled out using the standard fit of the UT (see [7] for further discussions).

After considering the implications of $S_{\pi\pi}$ on the UT, let's explore now $C_{\pi\pi}$. Since $C_{\pi\pi}$ is an odd function of ϕ, it is therefore sufficient to restrict the discussion to positive values of ϕ. A positive phase ϕ is obtained by the perturbative estimate in QCDF, neglecting soft phases with power suppression. For positive ϕ also $C_{\pi\pi}$ will be positive, assuming $\bar\eta > 0$, and a sign change in ϕ will simply flip the sign of $C_{\pi\pi}$.

In contrast to the case of $S_{\pi\pi}$, the hadronic quantities r and ϕ play a prominent role for $C_{\pi\pi}$, as can be seen in (4). This will in general complicate the interpretation of an experimental result for $C_{\pi\pi}$.

The analysis of $C_{\pi\pi}$ becomes more transparent if we fix the weak parameters and study the im-

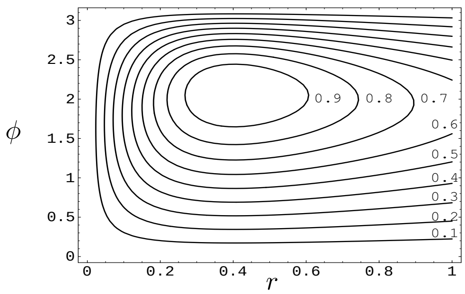

Figure 2. *Contours of constant* $C_{\pi\pi}$ *in the* $(r,\ \phi)$*-plane for the SM best-fit result* $(\bar\rho, \bar\eta) = (0.20, 0.35)$ *[11].*

pact of r and ϕ. An important application is a test of the SM, obtained by taking $\bar\rho$ and $\bar\eta$ from a SM fit and comparing the experimental result for $C_{\pi\pi}$ with the theoretical expression as a function of r and ϕ. In Fig. 2, a useful representation is obtained by plotting contours of constant $C_{\pi\pi}$ in the $(r,\ \phi)$-plane, for given values of $\bar\rho$ and $\bar\eta$. Within the SM this illustrates the correlations between the parameters (r, ϕ) and observable $C_{\pi\pi}$.

As it has been shown in [7], a bound on the parameter $C_{\pi\pi}$ exists, given by

$$C_{max} = \frac{2\kappa\,\sin\phi}{\sqrt{(1+\kappa^2)^2 - 4\kappa^2\,\cos^2\phi}}, \tag{9}$$

with $\kappa \equiv r/R_b$ and where the maximum occurs at $\cos\gamma = -2\kappa\cos\phi/(1+\kappa^2)$. If $\kappa = 1$, no useful upper bound is obtained. However, if $\kappa < 1$, then C_{max} is maximized for $\phi = \pi/2$, yielding the general bound $C < \frac{2\kappa}{1+\kappa^2}$. For the conservative bound $r < 0.15$, $\kappa < 0.38$ this implies $C_{\pi\pi} < 0.66$. The bound on $C_{\pi\pi}$ can be strengthened by using information on ϕ, as well as on κ, and employing (9). Then $\kappa < 0.38$ and $\phi < 0.5$ gives $C_{\pi\pi} < 0.39$.

4. Model Independent bound on the UT

As has been shown in [8], the following inequality can be derived from (8) for $-\sin 2\beta \le S_{\pi\pi} \le 1$

$$\bar\eta \ge \frac{1 + \tau S_{\pi\pi} - \sqrt{1 - S_{\pi\pi}^2}}{(1+\tau^2) S_{\pi\pi}}(1 + r\cos\phi). \tag{10}$$

This bound is still *exact* and requires no information on the phase ϕ.

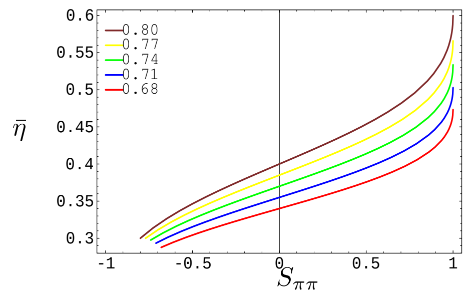

Figure 3. *Lower bound on* $\bar\eta$ *as a function of* $S_{\pi\pi}$ *for various values of* $\sin 2\beta$.

Assuming now $-90^\circ \le \phi \le 90^\circ$, we have $1 + r\cos\phi \ge 1$ and

$$\bar\eta \ge \frac{1 + \tau S_{\pi\pi} - \sqrt{1 - S_{\pi\pi}^2}}{(1+\tau^2) S_{\pi\pi}}. \tag{11}$$

We emphasize that this lower bound on $\bar\eta$ depends only on the observables τ and $S_{\pi\pi}$ and is essentially free of hadronic uncertainties. Since both r and ϕ are expected to be quite small, we anticipate that the lower limit (11) is a fairly strong bound, close to the actual value of $\bar\eta$ itself (see [7] for further details). We also note that the lower bound (11) represents the solution for the unitarity triangle in the limit of vanishing penguin amplitude, $r = 0$. In other words, the model-independent bounds for $\bar\eta$ and $\bar\rho$ are simply obtained by ignoring penguins and taking $S_{\pi\pi} \equiv \sin 2\alpha$ when fixing the unitarity triangle from $S_{\pi\pi}$ and $\sin 2\beta$. Let us briefly comment on the second solution for $\bar\eta$, which has the minus sign in front of the square root in (8) replaced by a plus sign. For positive $S_{\pi\pi}$ this solution is always larger than (8) and the bound (11) is unaffected. For $-\sin 2\beta \le S_{\pi\pi} \le 0$ the second solution gives a negative $\bar\eta$, which is excluded by independent information on the UT (for instance from ε_K).

Because we have fixed the angle β, or τ, the lower bound on $\bar\eta$ is equivalent to an upper bound on $\bar\rho = 1 - \tau\bar\eta$. The constraint (11) may also be expressed as a lower bound on the angle γ or a lower bound on R_t (see [7] for further details). In Figs. 3, we represent the lower bound on $\bar\eta$

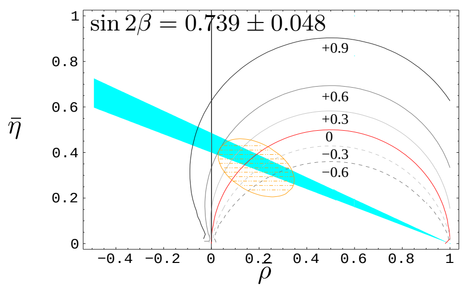

Figure 4. *Model-independent bound on the $(\bar{\rho}, \bar{\eta})$ plane from $\sin 2\beta$ (shaded area) and $S_{\pi\pi}$. The result of a standard UT fit (dotted ellipse, from [11]) is overlaid for comparison.*

as a function of $S_{\pi\pi}$ for various values of $\sin 2\beta$. From Fig. 3 we observe that the lower bound on $\bar{\eta}$ becomes stronger as either $S_{\pi\pi}$ or $\sin 2\beta$ increase.

In Fig. 4 we illustrate the region in the $(\bar{\rho}, \bar{\eta})$ plane that can be constrained by the measurement of $\sin 2\beta$ and $S_{\pi\pi}$ using the bound in (11). We finally note that the condition $r \cos\phi > 0$, which is crucial for the bound, could be independently checked [12] by measuring the mixing-induced CP-asymmetry in $B_s \to K^+K^-$, the U-spin counterpart of the $B_d \to \pi^+\pi^-$ mode [13].

5. Summary

In this talk, we have proposed strategies to extract information on weak phases from CP violation observables in $B \to \pi^+\pi^-$ decays even in the presence of hadronic contributions related to penguin amplitudes. Assuming knowledge of the penguin pollution, an efficient use of mixing-induced CP violation in $B \to \pi^+\pi^-$ decays, measured by $S_{\pi\pi}$, can be made by combining it with the corresponding observable from $B \to J/\psi K_S$, $\sin 2\beta$, to obtain the unitarity triangle parameters $\bar{\rho}$ and $\bar{\eta}$. The sensitivity on the hadronic quantities, which have typical values $r \approx 0.1$, $\phi \approx 0.2$, is very weak. In particular, there are no first-order corrections in ϕ. For moderate values of ϕ its effect is negligible.

Concerning our penguin parameters, namely r and ϕ, they were investigated systematically within the QCDF framework. To validate our theoretical predictions, we have calculate these parameters in the $1/m_b$ and $1/N_C$ expansion, which exhibits a good framework to control the uncalculable power corrections, in the factorization formalism. As an alternative proposition, we have also considered to extract r and ϕ from other B decay channels, such as $B^+ \to \pi^+\pi^0$ and $B^+ \to \pi^+K^0$, relying on the SU(3) argument. Using these three different approaches, we found a compatible picture in estimating these hadronic parameters.

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